How Do We Design for Concreteness Fading?
Survey, General Framework, and Design Dimensions

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ABSTRACT

Over the years, concreteness fading has been used to design learning materials and educational tools for children. Unfortunately, it remains an underspecified technique without a clear guideline on how to design it, resulting in varying forms of concreteness fading and conflicting results due to the design inconsistencies. To our knowledge, no research has analyzed the existing designs of concreteness fading implemented across different settings, formulated a generic framework, or explained the design dimensions of the technique. This poses several problems for future research, such as lack of a shared vocabulary for reference and comparison, as well as barriers to researchers interested in learning and using this technique. Thus, to inform and support future research, we conducted a systematic literature review and contribute: (1) an overview of the technique, (2) a discussion of various design dimensions and challenges, and (3) a synthesis of key findings about each dimension. We open source our dataset to invite other researchers to contribute to the corpus, supporting future research and discussion on concreteness fading.

CCS CONCEPTS

• **General and reference** → **Surveys and overviews**; • **Human-centered computing** → *Interaction design theory, concepts and paradigms.*

KEYWORDS

concreteness fading; literature review

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1 INTRODUCTION

Since the first *Interaction Design and Children* (IDC) conference in 2002, Piaget's theory of cognitive development [38], constructivism [36], scaffolding [84], and zone of proximal development [16] have been the most cited theories informing the design of interactions and technologies for children [37, 90]. While these theories differ in specifics, at high level they share a common belief in guiding children from concrete to abstract, or from known to unknown.

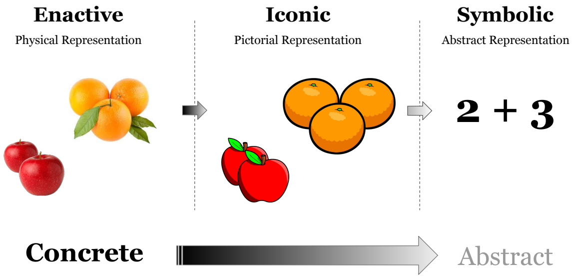

Figure 1: Bruner's framework for concreteness fading, a method for teaching abstract concept by introducing it in three stages with decreasing levels of concreteness [79].

Concreteness fading is an instructional technique that shares the same idea but specializes in the progression of concrete to abstract in representations, as shown in Fig. 1. As it is well-aligned with these established theories in developmental and educational psychology, it has been widely used to design learning materials and educational tools for children. In fact, over the years, research in mathematics and science education [27, 34] and recently in computing education [82] have shown its effectiveness with children.

Unfortunately, despite years of research, it remains an "underspecified" theory of instruction, without explicit design principles [29] and dimensions. Moreover, since the first conceptualization of the technique by Bruner [11], it has evolved with variants in the number of stages and types of representations used in each stage, making it difficult to know where to begin. Thus, to support future research on concreteness fading, we conducted a comprehensive literature review. Specifically, our research questions were:

- **RQ1**: What are the existing designs for concreteness fading?
- **RQ2**: What is the general framework we can derive from the previous designs and literature?
- **RQ3**: What are the design dimensions and issues to consider when designing it?

We provide an overview of concreteness fading, identify its design dimensions, and summarize the key findings about each dimension. Our analysis spans the research areas of math education [10, 54, 55], environmental health education [75], geography education [63], medical education [23], engineering education [83], science education [51, 71], and computing education [3, 43, 82]. In summary, our research contributes:

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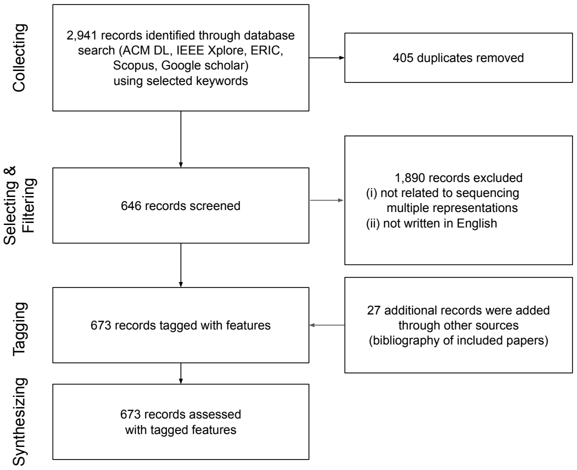

Figure 2: Flow diagram of our literature review process.

1. Analysis of a corpus of 219 papers, providing a comprehensive overview of concreteness fading and key findings related to the technique;
2. Design dimensions found across different implementations of concreteness fading; and
3. A dataset to support continued discussion and development of our understanding of concreteness fading.

2 METHODOLOGY

We conducted a systematic literature review following the standard procedure [44], as shown in Fig. 2. To arrive at a comprehensive collection of papers, we conducted a systematic search using five research databases associated with computing (ACM Digital Library and IEEE Xplore), education (ERIC), and multidisciplinary (SCOPUS, Google Scholar) areas. Our search terms included 'concreteness fading' as well as other variations of the concept, including 'concrete fading,' 'concrete to abstract,' 'progressive formalization,' 'progressive idealization,' 'multiple representation,' 'concrete-representational-abstract,' 'concrete-pictorial-abstract,' 'concrete-semiconcrete-abstract,' 'CRA,' 'RA,' 'CSA,' 'CPA,' and 'VRA.' After removing redundant papers, our dataset consisted of 2,536 papers.

Since our goal is to present a comprehensive analysis of concreteness fading technique, we did not limit our search to a certain time period, type of work, or original search query set. Thus, the publishing date of papers within our dataset ranges from 1966 to February 2019. We went through multiple stages in our filtering process. First, we assigned each paper a relevance tag: '0' for not relevant (i.e., off-topic); '1' for somewhat relevant (e.g., the paper is only peripherally related to concreteness fading); '2' for relevant (i.e., the study is about concreteness fading). We removed papers tagged with the relevance value of 0. This left us with 646 papers. Then, we went through multiple iterations of reading and tagging these papers, during which we added 27 additional papers by pursuing references of these papers (i.e., "snowballing" approach [89]) to end up with 673 papers. To analyze and document the design dimensions and implementation issues mentioned in each paper, we selected papers with relevance value of '2,' which gave us 219 papers. Within the papers tagged with relevance values of 1 and 2, we had a tag for each paper that indicated whether it was published at either conference or journal. We created this tag so that we could filter out papers that were not subject to review. However, we decided to retain them in order to (1) avoid publication bias, following the practice of some literature reviews [46], and (2) avoid excluding some of the works that have already been cited in other works.

Table 1: Number of papers published on concreteness fading between 1966 and 2019 across different disciplines

Domain	Number of papers
Math	177
Science	19
Computing	10
Others	13

We iteratively developed the tagging categories by conducting the thematic analysis [19]. Some of the final tagging categories (N=35) included design dimensions discussed in a review article by Fyfe [29], such as number of stages, and those found from analysis of our comprehensive review, such as within- versus across-lesson fading. We also included tagging categories related to the paper itself, such as contribution, and related to the experiments, such as participant age and sample size. The full list of our tagging categories and their descriptions are provided in our dataset.

As shown in Table 2, most research on concreteness fading comes in the form of empirical studies. Surprisingly, we found only one work [29] that voiced the need to develop design guidelines for concreteness fading. Witzel et al. [87] proposed seven steps that teachers can follow to create their own Concrete-Representational-Abstract (CRA) instructional sequence, but these were strictly limited to classroom settings for teaching math topics, and the first stage of the CRA sequence requires the use of a physical object.

Table 2: Classification of concreteness fading papers based on contribution types [88]

Contribution types	N	Description
Empirical	161	qualitative or quantitative data
Artifact	15	systems, tools, and environments
Methodological	1	methods
Theoretical	1	definitions, models, or frameworks
Dataset	0	new and useful corpus
Survey	19	review and synthesis of related work
Opinion	22	essays or arguments

3 OVERVIEW

Research on concreteness fading span many disciplines. Here, we summarize the major models that were implemented by prior studies in order to understand the variety of ways people have tried to implement concreteness fading.

Table 3: Different implementation structures & naming schemes of concreteness fading identified during our systematic literature review

	Stage 1	Stage 2	Stage 3
CRA [26, 52, 70, 78, 85, 86]	Concrete	Representational	Abstract
CPA [17, 50, 56, 61, 67]	Concrete	Pictorial	Abstract
CSA [39, 57, 59, 60, 63, 65]	Concrete	Semi-concrete	Abstract
VRA [6, 9, 66, 68]	Virtual	Representational	Abstract
CA [41, 69]	Concrete	-	Abstract
VA [7]	Virtual	-	Abstract
RA [12]	-	Representational	Abstract

3.1 Taxonomy

As shown in Table 3, different implementations and names of concreteness fading have emerged over the years, while maintaining the core idea of progressing from concrete to abstract in the delivery of a concept. We describe the major frameworks of concreteness fading in the literature, and the differences between them, if any.

The **Concrete-Representational-Abstract (CRA)** instructional framework, also referred to as Concrete-Semiconcrete-Abstract (CSA), uses the same representations as Bruner's: physical, pictorial, and abstract representations. An interesting feature of the CRA framework is that explicit instruction is embedded within the CRA sequence. In other words, at each stage of concreteness fading, teachers follow the three phases of explicit instruction: (i) modeling, (ii) guiding, and (iii) independence. In the modeling phase, teachers not only demonstrate how to solve the problems but also use think-aloud methods (i.e., narrating or verbalizing their thought process). In the guiding phase, students are given prompts or cues by teachers when they are stuck or make a mistake. Finally, in the independence phase, students solve the problems without any assistance [1, 8]. Typically, students do not proceed to the next stage in the CRA sequence (i.e., concrete to representational) until they achieve mastery in the independence phase, generally defined as an 80% success rate [8, 15]. As a result, the CRA instructional sequence often includes multiple lessons in a single stage. Many prior works on the CRA framework offer empirical support for its effectiveness for students with learning disabilities. These students typically have difficulties with abstract reasoning and problem-solving skills, which are both critical for success in mathematics [40]. The empirical support for the CRA framework in teaching students with learning disabilities is so robust that it is regarded as an evidence-based practice within math education [1, 8, 58].

Concrete-Pictorial-Abstract (CPA) is a term that was adopted when the framework became a key instructional method for the development of primary mathematics concepts in Singapore since the 1980s [50, 72]. While it adheres to Bruner's recommendation of mastering each stage before moving to the next one, teachers are not asked to follow a well-defined procedure and thus is different from the CRA framework. For example, in 2017, Chang Suo Hui et al. [17] proposed a CPA model that consists of four phases in each stage, whereas the CRA framework follows three phases of explicit instruction in each stage.

The **Virtual-Representational-Abstract (VRA)** is an adaptation of the CRA framework proposed by Bouck [7] in 2017. Bouck stated that what separates VRA from CRA is that virtual manipulative is used in place of physical object in the concrete stage. The remaining stages, i.e., representational and abstract stages, are the same as in the CRA framework. Although Bouck coined the term in 2017, she was not the first to use virtual manipulative within the CRA framework. Cooper [18] discussed virtual manipulative under the CRA framework in 2012, and several other studies also used virtual manipulative under the CRA framework [30, 47].

The **Concrete-Abstract (CA), Virtual-Abstract (VA), and Representational-Abstract (RA)** are two-stage progression sequences either without a concrete stage or an intermediate, representational stage (cf. Table 3). Like many others [29], we adopt a broad interpretation of Bruner's framework and accept these sequences to be variations of concreteness fading. In general, the two stage progression sequences can be an efficient form of concreteness fading for students who may not necessarily benefit from having either a concrete or intermediate stage before progressing to the abstract stage [15].

3.2 General Framework

Although concreteness fading has evolved over the years with different implementations, it has been without a generic formula to formalize the concept. Our analysis has shown that concreteness fading consists of a set of stages, where each stage can have a unique number of lessons. More formally, a concreteness fading framework consists of a set of stages s_i where $i = 1...S$ and $S \geq 2$. Each stage s_i contains a set of lessons $l_{i,j}$ where $j = 1...L_{s_i}$ and $L_{s_i} \geq 1$.

4 DESIGN DIMENSIONS

Our work fills an important gap in concreteness fading research, as no prior work has proposed a design space for concreteness fading through a comprehensive systematic literature review [87]. In this section, we explore specific design dimensions found in research on concreteness fading. Our intention is not to provide design guidelines, but to present the issues and design choices present in each dimension. These design dimensions include order of progression, number of stages, representation, presentation method, connection between representations, and within-lesson versus across-lesson fading. Note that they concern design components. Thus they do not include parameters, such as age and domain, that may be considered when making design choices but are not part of the design. However, these parameters are included in our dataset for reference.

4.1 Order of Progression

Typically, concreteness fading assumes that a concrete-to-abstract sequence leads to optimal learning. The argument for progressing from concrete to abstract is that it enables abstract concepts to be grounded in meaningful, familiar scenarios, while still guiding a generalization beyond the given context. Many studies [28, 82] tested this assumption by comparing it with other possible sequences, such as concrete-only, abstract-only, and abstract-to-concrete. We present the argument for and experiments related to these different progression mechanisms to provide an overview.

4.1.1 Only Abstract. Scholars that support keeping the representations abstract argue that concrete representations possess unnecessary perceptual details that interfere with students' understanding of the core concept. Dwyer [24] suggests that the excessive perceptual richness may evoke undesired responses that hinder the learning process. Other scholars suggest that students will end up learning the contextualized knowledge, but not learning the core concept [13, 22, 25, 33]. An experiment conducted by Kaminski et al. [42] showed that when teaching undergraduate students a complex mathematical concept, keeping the representation abstract led to greater learning than concreteness fading, providing empirical evidence for the only-abstract argument. Braithwaite and Goldstone [10] also reported similar results with teaching mathematics to undergraduate students. But since they were undergraduate students, who are capable of abstract reasoning according to cognitive development theory [37], these results are not surprising.

4.1.2 Abstract to Concrete ("Concreteness introduction"). Increasing the concreteness of a concept is closely related to the idea of tell-and-practice, a form of direct instruction [14, 45]. In this instructional setting, students are taught the abstract idea directly, and then are given concrete examples to practice. Spiro [77] has suggested that after the students have mastered the core concept, practicing with multiple concrete examples based on the concept would help them understand how the concept can be applied and transferred across different contexts. Johnson et al. [41] conducted an experiment by teaching middle school students electrical circuits, and results showed that students who learned through the abstract-to-concrete progression performed better than those who learned with two-stage implementation of concreteness fading.

4.1.3 Only Concrete. Tapola et al. [81] compared the performance of fifth- and sixth-grade students learning electrical circuits, comparing those who learned with the concreteness fading procedure and those who studied only using concrete examples. Students who received concrete examples only learned better, with Tapola suggesting that concrete examples enhance students' interest in a concept.

4.1.4 Concrete to Abstract ("Concreteness fading"). The theoretical support for moving from concrete to abstract is based on the cognitive development theory [31], which posits that cognitive development proceeds from concrete to abstract. While the findings from Kaminski et al. [42] and Johnson et al. [41] were used to argue that moving from abstract to concrete is the right sequence, Fyfe et al. [28] and Trory et al. [82] recently provided evidence that concrete to abstract is significantly better in three-stage progression. As the experiments by Fyfe et al. and Trory et al. employed a direct comparison where the only difference between the two conditions was the order in which they received instruction, they argued that the order of progression matter and that instruction should move from concrete to abstract.

4.2 Number of Stages

The number of stages determines how quickly a user is introduced to the symbolic representation of a given concept. We find that the majority of past concreteness fading implementations have three stages, as the major variations of concreteness fading, the CRA/CSA/CPA frameworks, are predicated on Bruner's three-stage progression model of concreteness fading.

4.2.1 Two Stages vs. Three Stages. There have been past attempts to test different numbers of stages of the concreteness fading technique, primarily two stages. One of the major driving factors for implementing two-stage progression was efficiency. If two-stage progression—without either the concrete or representational stage—could still lead to the same level of learning gains as the three stage progression, the technique would require fewer resources and less instructional management.

By this line of reasoning, Butler et al. [12] compared the learning gains of students with learning disabilities in the three-stage CRA sequence, to students in the two-stage RA sequence in middle school. They found that students assigned to three-stage CRA performed significantly better than those in the two-stage RA. Interestingly, however, both the CRA and RA (two stage progression without concrete manipulative) groups performed at least as well as their peers in the comparison group who had no learning disabilities. In other words, the CRA framework, even in its two-stage progression, enabled students with learning disabilities to perform just as well as those without learning disabilities.

Bouck et al. alo implemented two-stage progression [5] applying the Virtual-Abstract (VA) framework to students in sixth grade. In this framework, after students achieved some level of mastery with solving the fraction problems via virtual manipulative on the screen, they progressed to solving the problems with numbers (symbolic representation).

While there is empirical support for three-stage over two-stage progression, if one opts for two stage progression and faces the decision to remove either the concrete or representational stage, findings of the studies conducted thus far on the matter suggest that the intermediate representational stage may not be a vital component of the progression [15, 53].

4.2.2 Beyond Three Stage. In her article describing the concrete learning experience, Sowell [76] described the degree of abstractness in learning experiences and suggested that there are four stages: concrete, concrete-abstract, pictorial-abstract, and abstract. She asserted that when students begin to show understanding in the concrete stage, they should proceed to record what they are doing. She described this stage as the 'concrete-abstract' stage. Her reasoning was that while students take note of what they are doing in their activity, they may notice the underlying relationship in their activity and undergo abstraction learning at this stage. Sowell's 'concrete-abstract' stage, however, may not be generalizable, as it may be undesirable or unnecessary in certain contexts to incorporate this kind of student activity.

We did not find additional examples where researchers construct instructional sequence that have four or more stages. What we may infer from Sowell's case is that an additional stage may be added with particular activities during the progression.

4.3 Representation

In concreteness fading, representations at each stage must both differ and decrease in terms of their concreteness as stages progress. But what is concreteness? Fyfe [29] suggests that concreteness varies in terms of how idealized or generic a representation is from another representation. That is, it is not a dichotomous variable but rather a continuum where the point of reference is adjacent representation. She explains that a more idealized representation is closer to the core idea or invariant relation, whereas a less idealized representation adds more "concreteness" (i.e., information) in a way that makes spotting the core idea or invariant relation more difficult.

Under this view, the type of information or concreteness that is added to the representation can vary, as depictions can be considered less idealized on several dimensions. As such, there is not a single continuum, but many [29]. As long as the information added makes the representation less idealized, we can consider it as more "concrete" than another. Here, we present several concreteness types that have been used in implementations of concreteness fading in the literature. Note that this is not intended to be an exhaustive list of all possible concreteness types.

4.3.1 Concreteness in Physicality. Fyfe [29] suggests that the concrete-abstract spectrum of physicality pertains to whether the representation is 2-dimensional (e.g., a drawing on paper) or 3-dimensional (e.g., counting blocks). In the VRA framework, where the physical manipulative is replaced with a virtual manipulative, the virtual manipulative is typically similar to its physical counterpart except that it is introduced through a digital interface. Thus, even in VRA, one may argue that virtual manipulative still retains the physicality aspect, as can be seen from the work of Flores [26] where representations progress from 3-dimensional objects on screen to 2-dimensional objects, and then to symbolic representations.

4.3.2 Concreteness in Embodiment. Research in cognitive science has shown that having students use gestures or actions to simulate concepts can enhance their learning [32]. From this line of work emerged the *gesture hypothesis*, which speculates that iconic gestures (e.g., describing actions or objects with free-form gestures) should help learners embody mathematical concepts better than those using deitic gestures (e.g., pointing). In their concreteness fading experiment, Swart et al. [80] expanded on this notion of embodiment-based learning, and placed iconic and deitic gestures at the two ends of the concrete-abstract spectrum.

4.3.3 Concreteness in Concept Complexity. CTArcade [48] is a web application framework for teaching computational thinking skills that was designed based on the idea of concreteness fading. In its design, concept complexity is used as a measure of concreteness, moving students from concrete, simple concepts to abstract, complicated concepts.

4.3.4 Concreteness in Perceptual Richness. In her review on concreteness fading, Fyfe [29] situates external representations in the perceptual richness continuum on the basis of their color scheme. Representations with rich multiple colors and those with single, bland colors were placed at each end of the concrete-abstract spectrum. Although we did not come across any implementations in the literature that merely fade the color of a representation during the transition from one stage to another, many concreteness fading implementations [28] that fade from the concrete stage with a physical manipulative to the representational stage with a pictorial representation have colors stripped away from such pictorial representations.

4.3.5 Concreteness in Information. Fyfe [29] asserts that conceptual information refers to "the knowledge activated by the learner," and suggests familiarity of the representation and its narrative context as the two common types of conceptual information. A digital tablet game designed by Swart et al. [80] to teach fractions tested the concreteness fading approach with concrete (strong) and abstract (weak) narrative.

4.4 Presentation: Sequential vs Simultaneous

Concreteness fading is an instructional technique that suggests a particular way of presenting multiple external representations. While multiple representations can be introduced either sequentially or simultaneously, our analysis shows that all implementations of concreteness fading present each representation sequentially and never simultaneously. One benefit of simultaneous presentation is to allow an explicit mapping of representations to one another and more direct comparison. However, literature makes it clear that if the mapping is not supported, it does more harm than it helps [2, 21]. Fyfe [29] suggests that this sequential presentation approach is what differentiates concreteness fading from other approaches–that present multiple representations simultaneously and highlight the similarities and differences for mapping [62, 73]–and that it plays an important role in the fading progression by reinforcing the notion that the representations at each stage are mutual referents possessing the same set of invariant relations.

4.5 Connection between Representations

In instructional settings, it is recommended that instructors make explicit connection between representations that are more concrete and those that are more abstract, if the transfer of learning is to be successful [4, 29, 49, 64]. While this is a common point of emphasis in the literature, our analysis shows that some implementations do not explicitly inform or make the representations similar enough for learners to easily notice the connections between them [3].

A crucial aspect of the CRA framework is for teachers to explicitly inform students that the representations at each stage are connected to each other. This ensures that students do not miss such connections. However, in our analysis, we find several implementations where researchers do not explicitly mention to students the connection between given representations. This may be adequate if the representations are sufficiently similar to one another, but with some implementations it is hard to imagine that students would be able to notice that the sequential representations are mutual referents of the same concept [3]. In light of this, it is unsurprising that these experiments reported no significant improvement in learning through the concreteness fading technique, a result that conflicts with findings from most previous research. Failing to inform students of the connections between representations or otherwise making them explicit appeared to be the most common design mistake across implementations of concreteness fading. This finding adds further support for a common framework and an outline of design dimensions: they can inform the effective design, support comparison of different implementations, and in the process, enable systematic analysis of the technique.

4.6 Within-Lesson vs Across-Lesson Fading

We found in past literature that concreteness fading has been implemented either within a single lesson or across multiple lessons. In the latter case, for instance, students may be in a concrete stage where they interact with physical manipulative for several lessons until they progress to the representational stage. As was mentioned in Section 3, this is often seen in the CRA framework.

Whether to implement concreteness fading within a lesson or across multiple lessons may depend on the students and the difficulty of the given concept. As fewer number of lessons offers practical benefits, such as reduced time and effort required, knowing how many lessons are needed may be an important factor to consider when designing for concreteness fading. In replicating the previously conducted experiment with concreteness fading, Mancl et al. [52] reduced the number of lessons from 26 to 11 and saw that learners were able to achieve mastery within the reduced number of lessons. Recent implementations adopted within lesson progression and demonstrated that mastery is possible within a lesson [28, 82].

5 DISCUSSION

5.1 Expected Contribution to IDC Community

A prior work analyzing the first 10 years of research at IDC revealed that many researchers in the community are interested in designing technologies that help bridge the physical and digital. As Yarosh et al. [90] noted, this is further evidenced by the researchers at IDC encouraging the use of mixed reality (e.g., floor-based projection) and tangibles (e.g., tangible user interface), which represent two of the most common design choices in the community–the authors found this to be true in recent proceedings as well.

While concreteness fading focuses on linking the concrete and abstract *representations*, its theoretical framework can be useful in addressing this need. Notably, in addition to the proposed framework, the design dimensions and considerations for this technique may be applicable in designing technologies focused on bridging physical and digital. For instance, if we want children to transition from a tangible programming tool to its virtual representation on screen, we may question whether this progression will consist of two-stages or three-stages; for mixed reality, should the designer present physical and digital sequentially or simultaneously? Does the connection between the physical and digital need to be made explicit? If yes, by telling children explicitly or using visible cues? As shown, the range of design choices and associated issues for concreteness fading can afford a useful ground and reference for an investigation into designing technologies that bridge the physical and digital.

Surprisingly, concreteness fading is an underexplored technique at IDC. Except for one paper in 2018 [82] that used the technique to design an augmented reality-based learning environment for teaching a computing concept (internet routing), the authors did not find any paper from the IDC conference proceedings that used the technique. It is possible that some researchers at IDC are already familiar with this technique or do not know anything about it. Either way, we believe this work contributes a useful point of reference.

5.2 Limitations and Future Work

While we tried our best to find all research papers leveraging concreteness fading across many fields by using multiple databases and search terms, we may have missed some works that apply the core idea of concreteness fading to their designs but do not use the terminology associated with the technique or fail to cite the concreteness fading literature. Also, while a recent paper on touchscreen interaction design for children [74] constructed a design guideline regardless of conflicting findings in literature, we do not present a guideline with design recommendations. Instead, we outlined some general issues to consider when making design choices for each dimension. We did this because our analysis of the corpus revealed that findings related to the design of concreteness fading might be task-specific, and might also depend on variables like the age and skill levels of the students. Despite this limitation, we believe our analysis still supports the informed design decisions and contributes the foundation for us to continue to build on the findings of previous and future studies.

Indeed, our analysis has shown that there are many different implementations and naming schemes for concreteness fading. This shows a lack of communication and collective effort between disciplines and research communities to develop a shared understanding–which is not uncommon in research communities [35]. This often leads to researchers reinventing the work of other communities that have much to offer. This is unfortunate as it impedes scientific progress. A solution that was proposed from the CHI 2018 "Bridging HCI" workshop [20] was a standard interface through which researchers could easily exchange materials without discussion. Agreeing with this approach, we open source our dataset through a Github repository[1] so that we can continue to update our understanding of the technique with new findings and materials. The repository contains a description of our dataset and a link to a Google spreadsheet, which allows others to interact with the dataset. For instance, they can filter the dataset to see a set of studies conducted with primary school children or find a group of concreteness fading designs using 3-stage progression. We hope the repository serves as a standard interface through which researchers can learn about and exchange materials on concreteness fading.

6 CONCLUSION

Through a comprehensive literature review, we noticed how research on concreteness fading is fragmented and scattered across disciplines. Also, we found that lack of design guidelines and shared understanding of design dimensions produced conflicting results. We therefore formulated a common framework, and outlined the dimensions and design choices that need to be considered when designing for concreteness fading. While there have been past literature reviews [29] focusing on specific forms of concreteness fading, this is the first to analyze all of its variations to derive a general framework and elucidate design dimensions. We also open source our dataset to support future research on concreteness fading.

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[1] https://github.com/sanghosuh/concreteness_fading-dataset

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8 SELECTION AND PARTICIPATION OF CHILDREN

No children participated in this work.

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